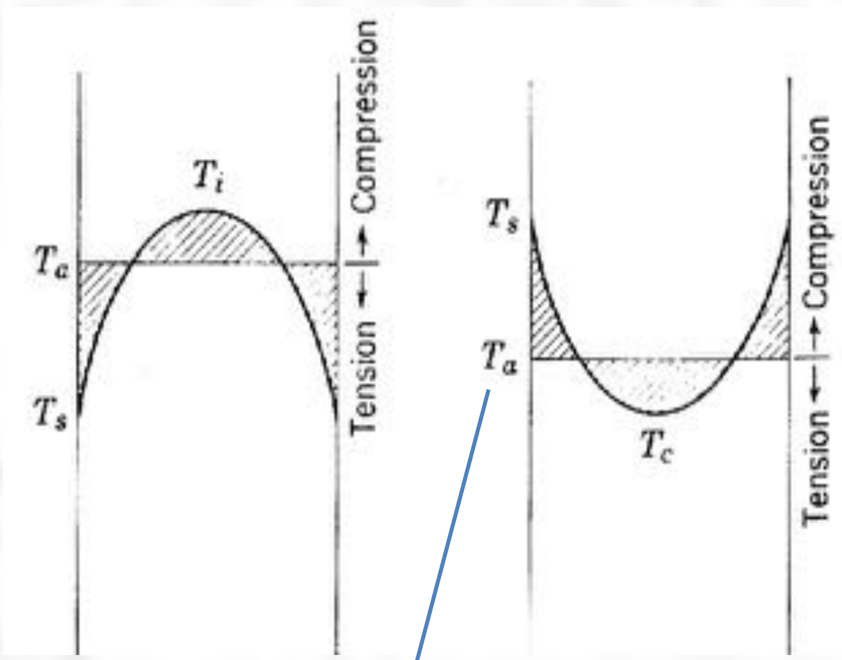
Introduction to ceramics, W. D. Kingery, H. K. Bowen and D. R. Uhlmann, J. Wiley & Sons, 1975 – Ch. 16

Annealing

□

→ **removal of residual stresses**

stresses due to thermal gradient in the component



cooling **heating**

average temperature

thermal expansion coefficient

Shape	Surface	Center
Infinite slab	$\sigma_z = 0$ $\sigma_y = \sigma_z = \frac{Ea}{1-\mu}(T_a - T_s)$	$\sigma_z = 0$ $\sigma_y = \sigma_z = \frac{Ea}{1-\mu}(T_a - T_c)$
Thin plate	$\sigma_y = \sigma_z = 0$ $\sigma_z = aE(T_a - T_s)$	$\sigma_y = \sigma_z = 0$ $\sigma_z = aE(T_a - T_c)$
Thin disk	$\sigma_r = 0$ $\sigma_\theta = \frac{(1-\mu)Ea}{1-2\mu}(T_a - T_s)$	$\sigma_r = \frac{(1-\mu)Ea}{2(1-2\mu)}(T_a - T_c)$ $\sigma_\theta = \frac{(1-\mu)Ea}{2(1-2\mu)}(T_a - T_c)$
Long solid cylinder	$\sigma_r = 0$ $\sigma_\theta = \sigma_z = \frac{Ea}{1-\mu}(T_a - T_s)$	$\sigma_r = \frac{Ea}{2(1-\mu)}(T_a - T_c)$ $\sigma_\theta = \sigma_z = \frac{Ea}{2(1-\mu)}(T_a - T_c)$
Long hollow cylinder	$\sigma_r = 0$ $\sigma_\theta = \sigma_z = \frac{Ea}{1-\mu}(T_a - T_s)$	$\sigma_r = 0$ $\sigma_\theta = \sigma_z = \frac{Ea}{1-\mu}(T_a - T_c)$
Solid sphere	$\sigma_r = 0$ $\sigma_t = \frac{Ea}{1-\mu}(T_a - T_s)$	$\sigma_t = \sigma_r = \frac{2Ea}{3(1-\mu)}(T_a - T_c)$
Hollow sphere	$\sigma_r = 0$ $\sigma_t = \frac{aE}{1-\mu}(T_a - T_s)$	$\sigma_r = 0$ $\sigma_t = \frac{aE}{1-\mu}(T_a - T_c)$

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for constant heating/ cooling velocity (Φ):

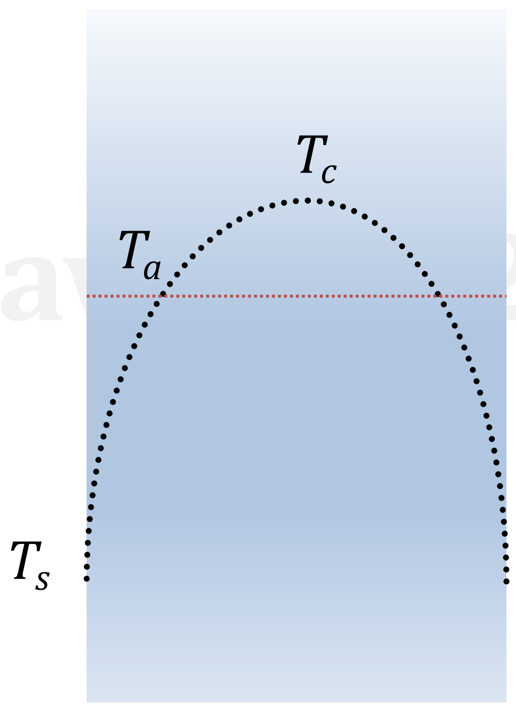
$$\sigma_s = \frac{E\,a}{1-\mu}\left(T_c - T_s\right)$$

viscosity at various temperatures
(possible stress relaxation)

where:

Shape	$T_c - T_s$
Infinite plate, half thickness = r_m	$0.50\ \frac{\phi r_m^2}{k/\rho c_p}$
Infinite cylinder, radius = r_m	$0.25\ \frac{\phi r_m^2}{k/\rho c_p}$
Cylinder, half length = radius = r_m	$0.201\ \frac{\phi r_m^2}{k/\rho c_p}$
Cube, half thickness = r_m	$0.221\ \frac{\phi r_m^2}{k/\rho c_p}$

T_c - T_a = 1/3 (T_c - T_s)



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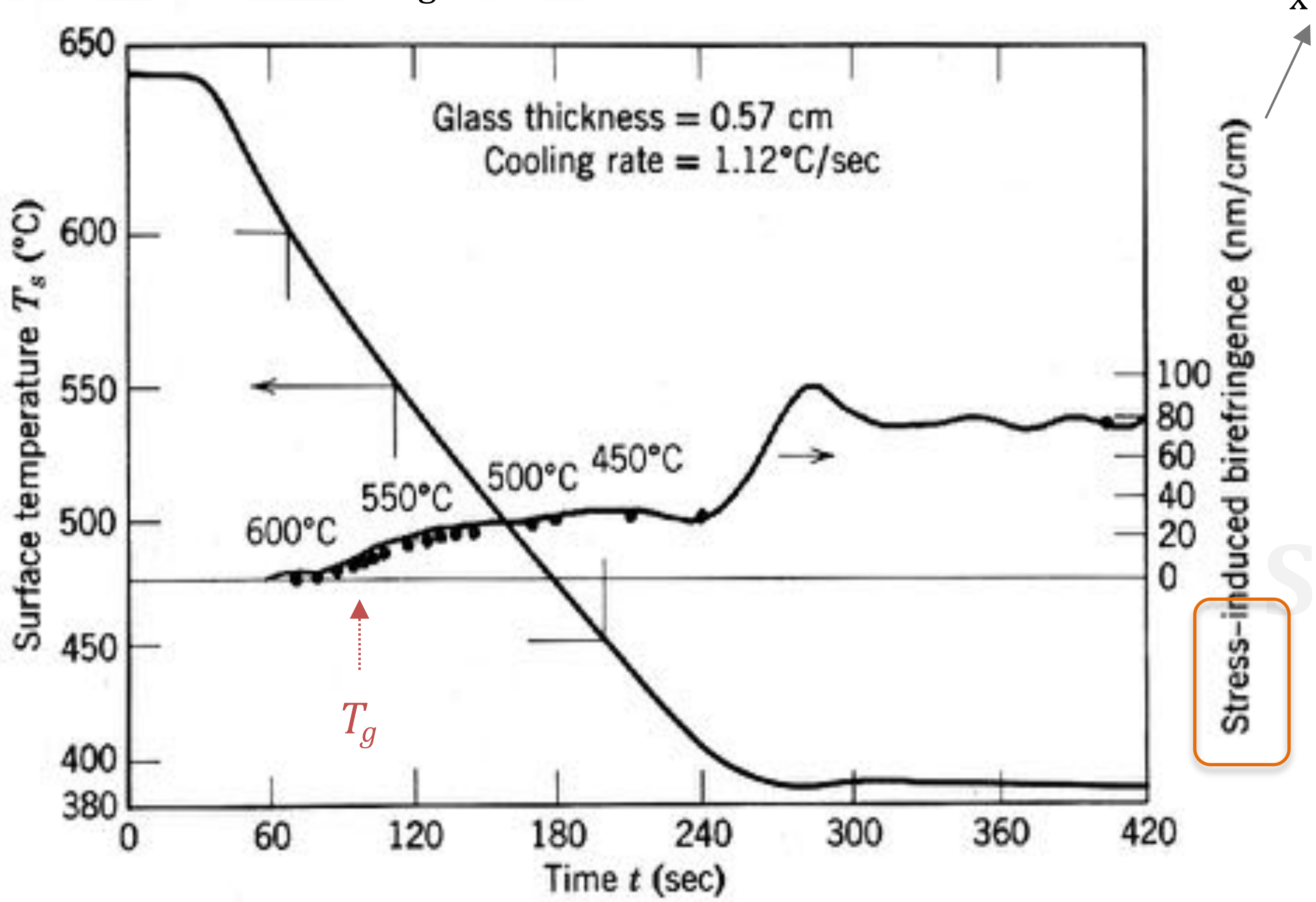
Example and additional effect (transition across T_g ÷ $\alpha_L \approx 3\ \alpha_S$)

soda-lime silicate glass

x 0.7 = MPa



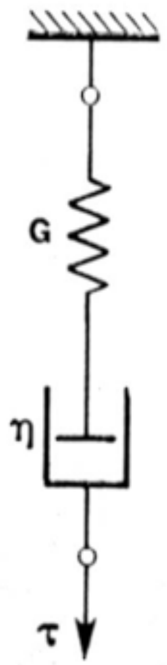
Annealing for stress relaxation

step 1: isotherm

Maxwell model:

$$\frac{d\sigma}{dt} = -\frac{1}{T_{rit}}\sigma \qquad T_{rit} = \frac{\eta}{G}$$

$$\sigma = \sigma_0 \exp\left(-\frac{t}{T_{rit}}\right)$$



***Adams & Williamson model** (empirical):*

$$\frac{d\sigma}{dt} = -A\sigma^2 \qquad \frac{1}{\sigma} - \frac{1}{\sigma_0} = At \qquad \sigma_0 \gg \sigma \qquad t \approx \frac{\exp(0.07(T_a - T))}{4.76x10^{-4}\,\sigma}$$

$$A = A_a \exp(0.07\,(T - T_a)) \qquad T \leq T_a$$

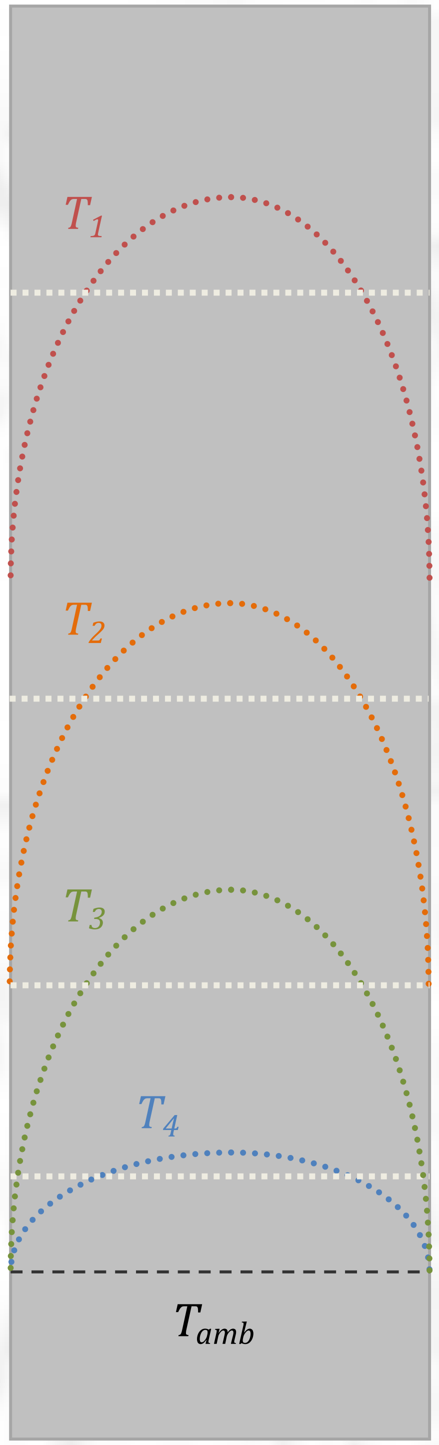
$$A_a = 4.76 \times 10^{-4}\ 1/\text{MPa s}$$

if σ = 2.5 MPa, $T = T_a$ → t = 14 min

annealing temperature ($T_{g,dil}$ - 10°C)

step 2: cooling

if Φ = costant → T profile remains costant



$$\Delta T = T_c - T_s = \frac{1}{2}\frac{\phi\, r_m^2}{k/\rho c_p}$$

t_1 $\sigma \approx 0$

t_2 $\sigma \approx 0$

t_3 $\sigma \approx 0$

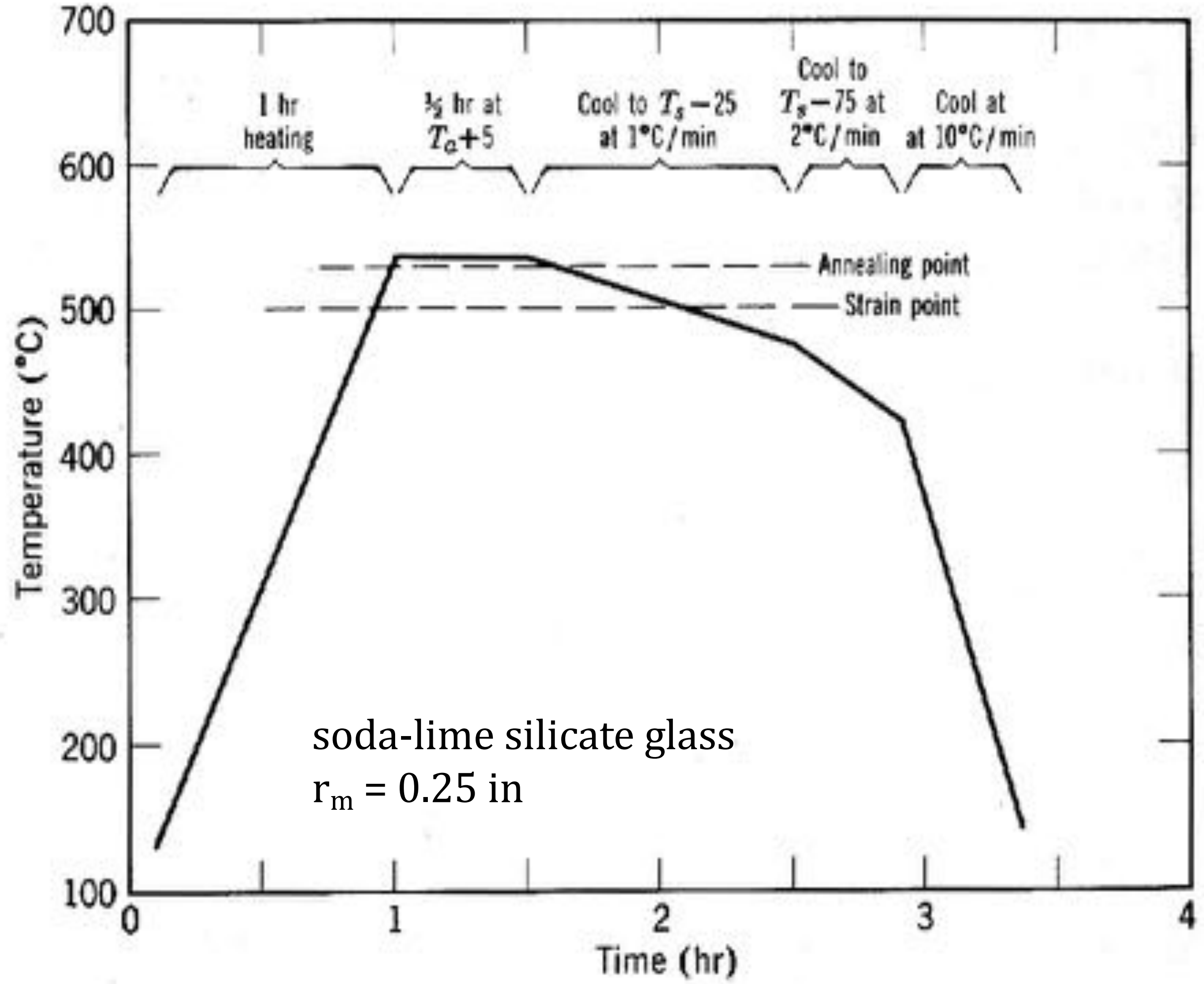
t_4 $\boldsymbol{\sigma \neq 0}$

$$\sigma_t = \frac{E\,a}{1-\mu}\left(T_c - T_a\right) = \frac{1}{6}\frac{E\,a}{\left(1-\mu\right)}\frac{\phi\, r_m^2}{k/\rho c_p}$$

$a = 9 \times 10^{-6}$ °C^{-1}
E = 70 GPa
r_m = 1 cm
σ_t = 2.5 MPa
ν = 0.3
$k/\rho\, c_p$ = 0.008 cm^2/s

→ Φ = 0.13 °C/min

typical annealing schedule



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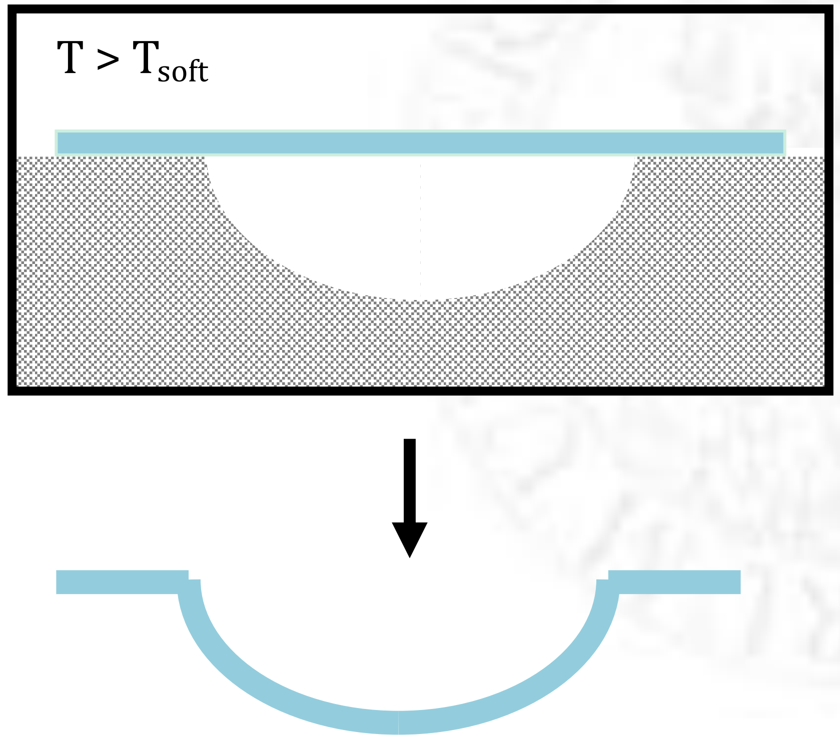
Successive operations

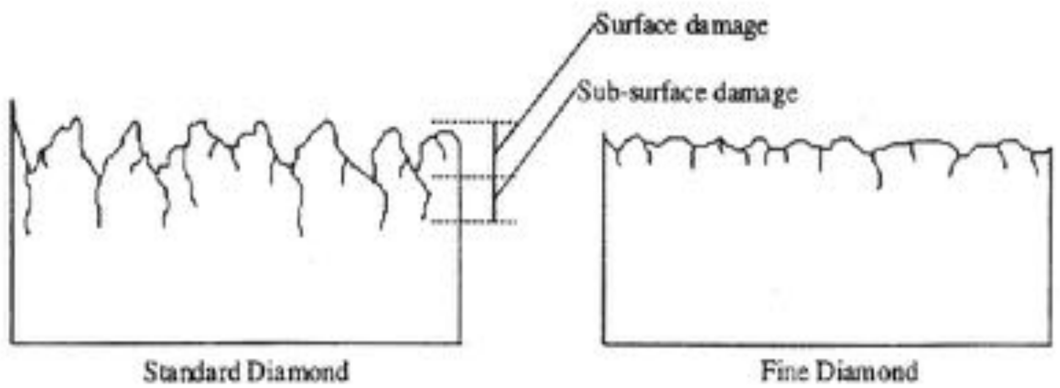
- **Tempering**
- **Lamination**

} *superior mechanical properties*

- **Softening & shaping ("fusion")**
- **Cutting, machining, polishing**



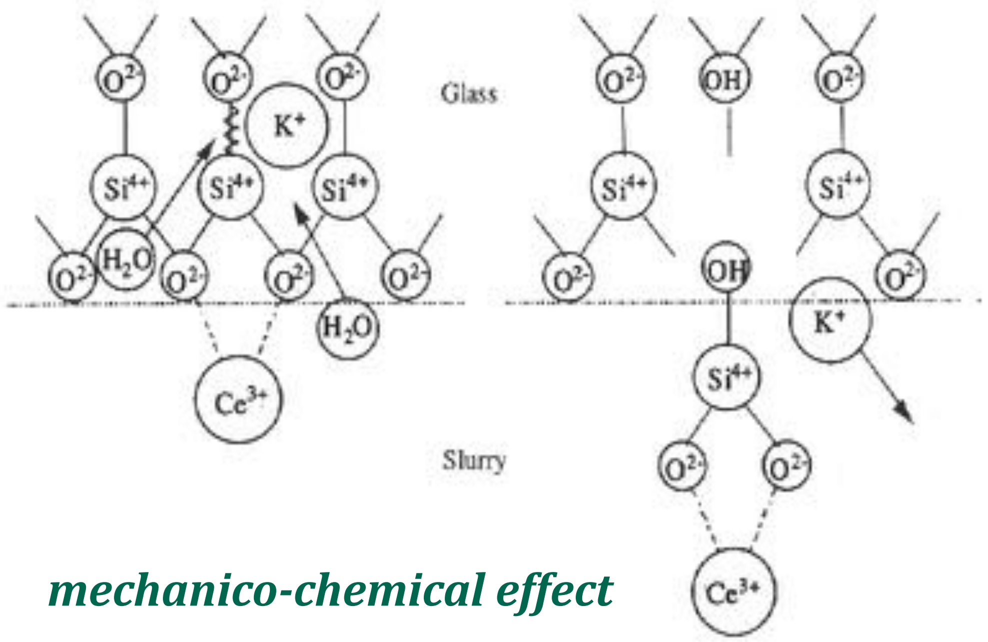
➢ diamond tools



mechanical effect

➢ diamond tools

➢ cloth + CeO_2, ZrO_2, Fe_2O_3



mechanico-chemical effect

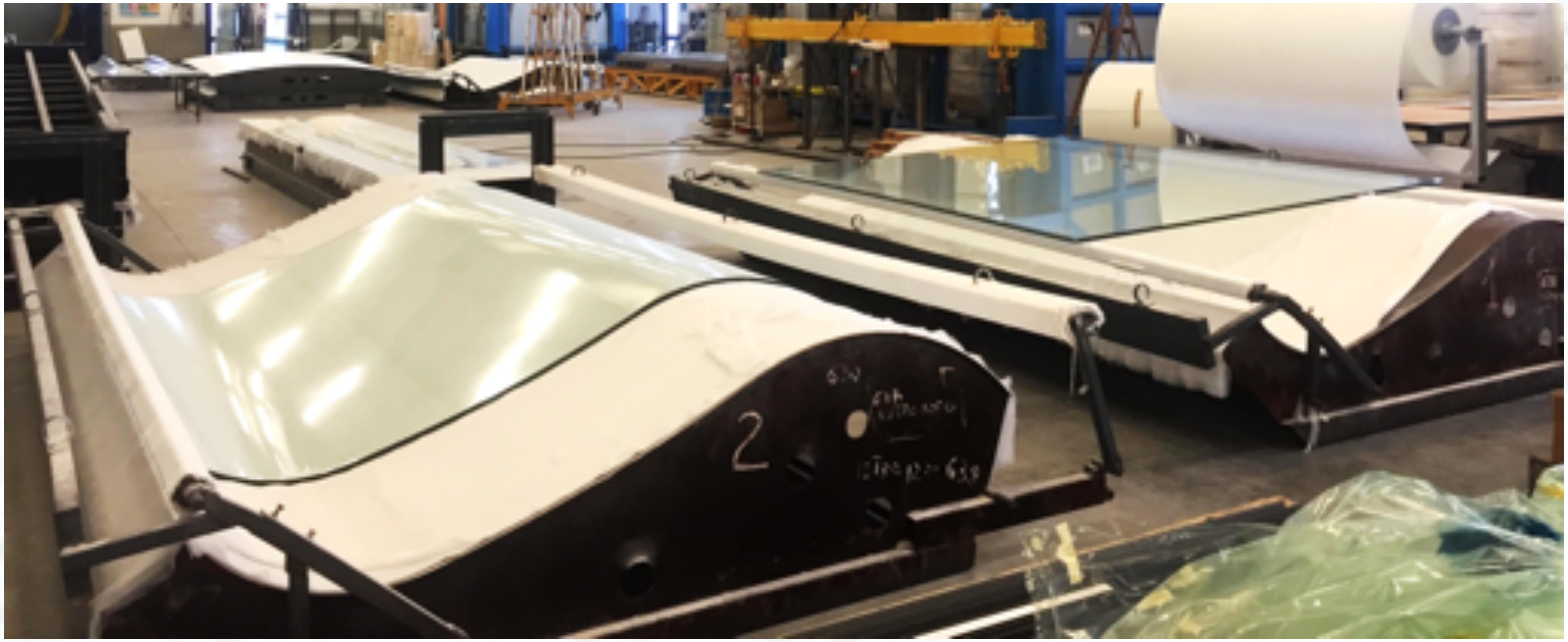
➢ diamond wheels or tools

➢ blades

➢ laser

➢ water-jet (pressurized water + abrasive)

A. Kelsall, Glass technology , 39, 1, 1998



- treatments in HF e NH_4F solutions

- **Surface etching for glossy effect ("frosting", "ghiacciatura", "satinatura")**
- **Coating** → *superior optical and thermal properties*
- **Decoration**

- spray, brush, ink-jet printing (polymers emulsions) + heating
- spray, brush, dipping, screen-printing, ink-jet printing (glass powder suspension) + heating/sintering